

Dynamics and properties of chiral cosmic strings in Minkowski space

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Chiral cosmic strings are produced naturally at the end of inflation in supersymmetric models where the symmetry is broken via a D term. Consequently in such theories, where both inflation and cosmic strings contribute to the density and CMBR (microwave background) perturbations, it is necessary to understand the evolution of chiral cosmic string networks. We study the dynamics of chiral cosmic strings in Minkowski space and comment on a number of differences with those of Nambu-Goto strings. To do this we follow the work of Carter and Peter who showed that the equations of motion for chiral cosmic strings reduce to a wave equation and two constraints, only one of which is different from the familiar Nambu-Goto constraints. We study chiral string loop solutions consisting of many harmonics and determine their self-intersection probabilities, and comment on the possible cosmological significance of these results.

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I. INTRODUCTION

In the last few years many high accuracy calculations have been made of the cosmological consequences of Nambu-Goto (NG) cosmic strings [1–4]. Indeed, such predictions were recently compared to the Boomerang data [5–7]. There are good reasons why most studies of the cosmological effects of topological defects (see [8,9] for a summary) have concentrated on NG strings: these are the simplest type of cosmic string, and their equations of motion (at least in Minkowski space) can be solved exactly. On a lattice one can use the highly efficient Smith-Vilenkin algorithm [10], and in fact some of the recent predictions are based on Minkowski space codes of NG network evolution [3,7].

However, NG cosmic strings, of which the simplest example are the strings formed in the Abelian Higgs model, are not likely to be the most realistic type of cosmic string. Cosmic strings that can create significant density perturbations require grand unified theory (GUT) scale physics. If the Higgs field that forms the string couples to fermions in the GUT theory—as might well be expected of a Higgs field—then these fermions yield zero modes in the core of the string [11], thereby generating a current (which need not be electromagnetically coupled) along the string.

In this paper we are interested in chiral cosmic strings which arise naturally in supersymmetric (SUSY) theories [12], where a $U(1)$ symmetry is broken with a Fayet-Iliopoulos D term, resulting in a single fermion zero mode which travels in only one direction along the string—this

defines a chiral string. In the cosmological context, chiral strings are automatically formed at the end of inflation in SUSY models with a D term [13]. Furthermore, the zero mode (or chiral nature of the string) survives the subsequent supersymmetry breaking phase transition [14], and consequently both inflation and chiral cosmic strings contribute to the density and cosmic microwave background radiation (CMBR) perturbations in this scenario. Calculations of these observable predictions were carried out recently [7,15,16], and there the C_I 's were decomposed as

$$C_I = \alpha C_I^{\text{inflation}} + (1 - \alpha) C_I^{\text{NG strings}}$$

with $0 \leq \alpha \leq 1$. However, we believe that the use of the C_I from NG strings in the above formula is an oversimplification at least in the inflation plus chiral cosmic string scenario mentioned above. In the case of chiral cosmic strings, the presence of the fermion zero mode is likely to have a significant effect on the dynamics of the string network, which could therefore evolve very differently to a NG network. Indeed the action describing the evolution of chiral cosmic strings is very different from the NG action [17]. The purpose of this paper is to quantify some of the differences between the evolution of these two different types of cosmic string network.

Our starting point is the action for chiral cosmic strings first proposed in [17]. As usual, in order to derive the equations of motion from this action, gauge choices must be made: in Minkowski space with suitable gauge choices the equations of motion reduce to the remarkably familiar form given by [17]

$$\frac{\partial^2 \mathbf{x}}{\partial t^2} - \frac{\partial^2 \mathbf{x}}{\partial \sigma^2} = 0 \Rightarrow \mathbf{x}(t, \sigma) = \frac{1}{2} [\mathbf{a}(t + \sigma) + \mathbf{b}(t - \sigma)],$$

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$$\dot{\mathbf{a}}^2 = 1,$$

$$\dot{\mathbf{b}}^2 \leq 1,$$

where for instance $\dot{\mathbf{a}}(q) \equiv d\mathbf{a}(q)/dq$. These can be recognized as the usual NG equations of motion [8] with the only difference being the constraint on the derivative of \mathbf{b} which must now lie *within* the Kibble-Turok sphere rather than on it. We show in Sec. II B that the physical reason for this stems from the conserved charge on chiral strings: if this charge is zero, then $\dot{\mathbf{b}}^2 = 1$ and one is left with NG strings as required.

If the charge on the strings is maximal, then $\dot{\mathbf{b}}^2 = 0$. As we show in Sec. III, this latter special case is very interesting since it implies that $\dot{\mathbf{x}} = \mathbf{x}'$ with $|\dot{\mathbf{x}}| = 1/2$ so that the strings move along themselves at half the speed of light and never change shape or self-intersect. In the case of loops, this solution corresponds to arbitrary-shape stable vortons. For infinite strings it means that self-intersections (which produce loops) never occur. Since this is an important mechanism of energy loss in the case of NG strings, this result already gives an indication that the evolution of chiral and NG cosmic string networks may be very different. For general chiral string solutions, self-intersection is certainly possible, but we may expect that the probability is lower than in the NG case. To quantify the differences between NG and chiral string evolution, we study in Sec. III the self-intersection probability of loops with different numbers of harmonics on them and different conserved charges.

The plan of this paper is the following. For pedagogical reasons, we begin in Sec. II A by briefly reviewing the work of Carter and Peter (CP) [17]. For chiral strings it is not possible to impose exactly the same gauge conditions as are usually chosen for NG strings. In Sec. II B we review the possible gauge choices, in particular those made by Martins and Shellard [18] and by CP. We follow the latter, whose choice leads to very simple equations of motion, which are almost identical to those of the NG string and which, most importantly, are exactly integrable in Minkowski space. In Sec. III we use these simple equations to discuss general properties of chiral strings as well as the self-intersection properties of loops with different numbers of harmonics and different charges. Finally, our conclusions and plans for future work are discussed in Sec. IV.

Note that while we were trying to extend the results presented here to Friedmann-Robertson-Walker (FRW) universes, a paper by Blanco-Pillado *et al.* [19] appeared which also obtains the equations of motion above, though from a rather different point of view. Here we follow more closely the work of CP [17], and extend both of these papers to study some general properties of chiral cosmic strings and the self-intersection of loops.

II. CHIRAL AND NAMBU-GOTO STRINGS

A. Action and equations of motion

For pedagogical reasons, we here review briefly the work of Carter and Peter [17].

The action for chiral cosmic strings they proposed involves a dimensionless scalar field ϕ which can be interpreted as the phase of the current carriers condensed on the string. The action is [17]

$$S = - \int d^2\sigma \sqrt{-\gamma} \left(m^2 - \frac{1}{2} \psi^2 \gamma^{ij} \phi_{,i} \phi_{,j} \right), \quad (1)$$

where γ_{ij} ($i, j = \{0, 1\}$) is the induced metric on the world sheet:

$$\gamma_{ij} = x_{,i} \cdot x_{,j} \equiv g_{\mu\nu} x_{,i}^{\mu} x_{,j}^{\nu}.$$

Here $g_{\mu\nu}$ is the background metric, with signature $(+, -, -, -)$ (which in the following we shall take to be the Minkowski metric) and $x^{\mu}(\sigma^0, \sigma^1)$ denotes the position of the string at world-sheet coordinates σ^i . The first term in Eq. (1) is just the NG action for a string with tension m^2 .

The action (1) is invariant under reparametrizations, $\sigma^i \rightarrow \tilde{\sigma}^i = \tilde{\sigma}^i(\sigma^j)$, and also under transformations of ϕ , with a compensating transformation of ψ :

$$\phi \rightarrow \tilde{\phi}(\phi), \quad \text{with} \quad \psi \rightarrow \tilde{\psi} = \left(\frac{d\tilde{\phi}}{d\phi} \right)^{-1} \psi. \quad (2)$$

These freedoms must be removed by making gauge choices, as discussed below.

The dimensionless Lagrange multiplier ψ sets the constraint

$$\gamma^{ij} \phi_{,i} \phi_{,j} = 0 \quad (3)$$

which ensures that the cosmic strings are indeed chiral. Generally, current-carrying strings are characterized by two currents and their corresponding charges [9]. One of these currents, proportional to $\gamma^{ij} \phi_{,j}$, is conserved by virtue of the equations of motion; the other, proportional to $\epsilon^{ij} \phi_{,j}$, is topologically conserved. However, for chiral strings, because of Eq. (3), the two currents coincide, and so therefore do the two corresponding charges Z and N .

In Minkowski space, the equations of motion following from Eq. (1), in addition to Eq. (3), are

$$\partial_i (\sqrt{-\gamma} \psi^2 \gamma^{ij} \phi_{,j}) = 0 \quad (4)$$

and finally

$$\partial_i \left[\sqrt{-\gamma} \left(\gamma^{ij} + \frac{\psi^2}{m^2} \phi^i \phi^j \right) x_{,j}^{\mu} \right] = 0. \quad (5)$$

In 1+1 dimensions, a scalar field whose gradient is everywhere null is necessarily harmonic; i.e., Eq. (3) implies

$$0 = \nabla^j \nabla_j \phi = \frac{1}{\sqrt{-\gamma}} \partial_i (\sqrt{-\gamma} \gamma^{ij} \phi_{,j}),$$

which is consistent with Eq. (4) only if ψ is a function of ϕ , $\psi = \psi(\phi)$. We shall verify this later in particular coordinate systems.

From Eq. (4), we may provisionally define the current as $j^i = \psi^2 \phi^i$, which is conserved and null, satisfying

$$j_{ij}^i = 0.$$

However, this expression is *not* invariant under the ϕ transformation (2). There is an ambiguity in the definition of j^i : any current of the form $j^i = f(\phi) \phi^i$ is null and conserved; there is an infinity of conservation laws. This is a peculiarity of null currents in 1+1 dimensions. For definiteness, we choose the invariant current

$$j^i = \psi \phi^i. \quad (6)$$

B. Gauge choices

To proceed further, gauge choices must be made. The action (1) is reparametrization invariant—we can replace σ^0 and σ^1 by any functions of these variables. Moreover, we can replace ϕ by any function of itself, provided we change ψ to compensate as in Eq. (2).

For the NG string it is usual to choose the conformal gauge in which $\gamma_{ij}(\sigma^k) = \Omega(\sigma^k) \eta_{ij}$, with $\eta_{ij} = \text{diag}(1, -1)$. Explicitly, if we write $\tau = \sigma^0$, $\sigma = \sigma^1$, and denote derivatives with respect to τ and σ by an overdot and prime respectively, the conformal gauge is specified by two conditions

$$\dot{x}^2 + x'^2 = 0 \quad (7)$$

and

$$\dot{x} \cdot x' = 0. \quad (8)$$

This implies that the string's velocity is perpendicular to its tangent vector. For the NG string, because of the conformal invariance of the action, these two conditions are not in fact independent, and therefore do not fully specify the coordinates. We can in addition impose the temporal gauge condition

$$\tau = t \equiv x^0. \quad (9)$$

For chiral strings, however, these three conditions are inconsistent, so different choices are needed.

We first recall the choices made by Martins and Shellard (MS) [18], who studied a number of properties of chiral cosmic string loops. They opted to maintain the temporal gauge condition (9). As noted above, there is no longer freedom to choose the full conformal gauge as well. Instead MS chose the world-sheet metric to be diagonal, maintaining Eq. (8) but not Eq. (7). On defining $\epsilon^2 = (\sqrt{-\gamma} \gamma^{00})^2 = \mathbf{x}'^2 / (1 - \dot{\mathbf{x}}^2)$, it follows from Eq. (3) that $\phi'^2 = \epsilon^2 \dot{\phi}^2$ so that Eq. (4) yields

$$\partial_t[\psi^2 \phi'] = \partial_\sigma[\psi^2 \dot{\phi}],$$

confirming the general result that $\psi = \psi(\phi)$. MS chose to fix ϕ by setting $\psi^2 = \text{const} = 1$. The equations of motion following from Eq. (5) are then given by [18]

$$[\epsilon(1 + \Phi)]' = \Phi'$$

$$\epsilon(1 + \Phi)\ddot{\mathbf{x}} = \left[(1 - \Phi) \frac{\mathbf{x}'}{\epsilon} \right]' + \Phi \mathbf{x}' + 2\Phi \dot{\mathbf{x}}' \quad (10)$$

where $\cdot = d/dt$ and $' = d/d\sigma$ and $\Phi = \dot{\phi}^2 / (m^2 \gamma_{00})$. Loop solutions to these equations were studied in [18,20].

Since these gauge choices lead to rather complicated equations of motion, we shall opt instead to follow the paper of Carter and Peter [17] and choose one of the world-sheet coordinates to be proportional to ϕ . That is, choose $\eta = m^{-1} \phi$ (the factor of m^{-1} is introduced for dimensional reasons) to be one world-sheet coordinate and denote the second by q . By Eq. (3) this implies

$$\gamma^{\eta\eta} = 0 \quad \Rightarrow \quad \gamma_{q\eta} = 0. \quad (11)$$

Thus the line element on the world sheet is

$$ds^2 = A d\eta^2 + 2\Omega dq d\eta$$

where

$$\Omega \equiv \gamma_{\eta q} = \sqrt{-\gamma} x_{,\eta} \cdot x_{,q}$$

and

$$A \equiv \gamma_{\eta\eta} = x_{,\eta} \cdot x_{,\eta}. \quad (12)$$

With this choice of coordinates, we see again that $\psi = \psi(\phi)$ since Eq. (4) gives

$$0 = \partial_q [\sqrt{-\gamma} \gamma^{q\eta} m \psi^2] = m \partial_q [\psi^2] \Rightarrow \psi = \psi(\phi).$$

We also note from Eq. (6) that

$$j_\eta = m \psi, \quad j_q = 0 \Rightarrow j^\eta = 0, \quad j^q = \frac{m \psi}{\Omega}. \quad (13)$$

Now, the equation of motion (5) gives

$$2\partial_q \partial_\eta x^\mu + \partial_q [F(\partial_q x^\mu)] = 0 \quad (14)$$

where

$$F = (\Omega \gamma^{qq} + \psi^2 \gamma^{q\eta}) = \frac{1}{\Omega} (\psi^2 - A).$$

It is now clear that we can further simplify Eq. (14) by choosing the second coordinate q in such a way that $F=0$. Happily this is a consistent choice because then $A = x_{,\eta} \cdot x_{,\eta} = \psi^2$ should be a function of ϕ only, independent of q . But this is indeed the content of the simplified equation of motion (14) with $F=0$, namely

$$\partial_q \partial_\eta x^\mu = 0. \quad (15)$$

The simplicity of this equation shows the convenience of this gauge choice, in which both η and q are characteristic coordinates. The general solution of the equation of motion is

$$x^\mu(q, \eta) = \frac{1}{2} [a^\mu(q) + b^\mu(\eta)],$$

exactly as for the NG string.

Within the gauge choices so far made, we still have freedom to transform each of the coordinates η and q separately: $\eta \rightarrow \tilde{\eta}(\eta)$ and $q \rightarrow \tilde{q}(q)$. It is convenient to choose them so that $\eta = a^0$ and $q = b^0$, and hence $t \equiv x^0 = \frac{1}{2}(\eta + q)$. This is essentially a temporal gauge. Finally, let

$$q = t + \sigma, \quad \eta = t - \sigma.$$

Then the equation of motion (15) and constraints (12) and (11) reduce to

$$\begin{aligned} \ddot{\mathbf{x}} - \mathbf{x}'' &= 0 \Rightarrow \mathbf{x}(t, \sigma) = \frac{1}{2}[\mathbf{a}(q) + \mathbf{b}(\eta)], \\ \left(\frac{d\mathbf{a}}{dq}\right)^2 &\equiv \dot{\mathbf{a}}^2 = 1, \\ \left(\frac{d\mathbf{b}}{d\eta}\right)^2 &\equiv \dot{\mathbf{b}}^2 \leq 1, \end{aligned} \quad (16)$$

where $\dot{} = d/dt$ and $' = d/d\sigma$. Notice that Eqs. (16) resemble very closely the NG equations and constraints in the temporal, conformal gauge: the only difference is that now \mathbf{b} is constrained to lie *within* the Kibble-Turok sphere rather than on it. The physical reason for this will be discussed below.

We note that Blanco-Pillado *et al.* [19] made essentially the same gauge choice, though without introducing the Lagrange-multiplier variable ψ .

The meaning of the coordinate σ can be understood by constructing the stress energy tensor. With the gauge choices made above this is given by

$$T^{\mu\nu}(t, \mathbf{y}) = m^2 \int d\sigma (\dot{x}^\mu \dot{x}^\nu - x^{\mu'} x^{\nu'}) \delta^3(\mathbf{y} - \mathbf{x}(t, \sigma)),$$

which is formally identical to the NG stress energy tensor in the conformal-temporal gauge. Since

$$T^{00}(t, \mathbf{y}) = m^2 \int d\sigma \delta^3(\mathbf{y} - \mathbf{x}(t, \sigma))$$

is conserved in Minkowski space, it follows that σ again measures the energy or “invariant length” along the string.

Finally, we examine the reason for the inequality $\dot{\mathbf{b}}^2 < 1$, which follows from the conserved charge on the string. From Eq. (13) the physical current on the string is given by

$$j^t = j^\sigma = \frac{m\psi}{2\Omega},$$

so that the conserved charge is

$$N = Z = \int d\sigma \sqrt{-\gamma} j^t = \frac{1}{2} \int d\sigma m \psi.$$

Now, let

$$\dot{\mathbf{b}}^2 = k^2,$$

so that $\psi^2 = A = x_{,\eta} \cdot x_{,\eta} = \dot{\mathbf{b}}^2/4 = (1 - k^2)/4$, from which it clearly follows that $k^2 \leq 1$. Note that

$$N = \frac{m}{4} \int d\sigma \sqrt{1 - k^2}, \quad (17)$$

so the value of k determines the charge on the string: this takes its maximum value when $k=0$ everywhere (as we will see below this corresponds to interesting vorton solutions), and $N=0$ when $k \equiv 1$, which is exactly the NG limit as required.

In the next section we study the self-intersection properties of loops with different values of N .

III. PROPERTIES OF CHIRAL COSMIC STRINGS AND LOOP SELF-INTERSECTIONS

In this section we first describe some general properties of chiral cosmic strings which follow from Eqs. (16). Loop self-intersections are then studied.

A. General properties

From Eqs. (16), the velocity and tangent vectors of the string are given by

$$\dot{\mathbf{x}}(t, \sigma) = \frac{1}{2}[\dot{\mathbf{a}} + \dot{\mathbf{b}}], \quad \mathbf{x}'(t, \sigma) = \frac{1}{2}[\dot{\mathbf{a}} - \dot{\mathbf{b}}]. \quad (18)$$

Here $|\dot{\mathbf{a}}| = 1$, while $|\dot{\mathbf{b}}| = k \leq 1$. We shall generally assume that there is a nonzero current, so that $k < 1$. It then follows that chiral current-carrying cosmic strings in Minkowski space cannot have zero velocity. (Thus stationary loops do not exist for example.) Similarly the tangent vector of the string never vanishes either so that there are no cusps on these strings.

From Eqs. (18) it also follows that

$$\dot{\mathbf{x}} \cdot \mathbf{x}' = \frac{1}{4}[1 - k^2] > 0,$$

so that the velocity of a point on the string is not perpendicular to its tangent vector. Notice that this result is due to the gauge conditions we have chosen: with the gauge choice of MS, $\dot{\mathbf{x}} \cdot \mathbf{x}' = 0$.

Observe also that when $k=0=|\dot{\mathbf{b}}|$, Eqs. (18) imply that $\dot{\mathbf{x}} = \mathbf{x}'$. Since their velocity is always parallel to the tangent vector, these strings do not change their shapes, and thus never self-intersect. Furthermore, given that when $k=0$, $\dot{\mathbf{x}}^2 + \mathbf{x}'^2 = \frac{1}{2}$ and $\dot{\mathbf{x}} \cdot \mathbf{x}' = \frac{1}{4}$, it follows that, for $k=0$,

$$|\dot{\mathbf{x}}| = \frac{1}{2} = |\mathbf{x}'|.$$

Thus the strings move at half the speed of light. Recall from Eq. (17) that when $k=0$ the strings carry the maximal charge.

B. Loops

We now turn to the properties of loops which must satisfy the periodicity conditions

$$\mathbf{x}(t, \sigma + L) = \mathbf{x}(t, \sigma).$$

From Eqs. (18), this implies, in the center-of-mass frame,

$$\mathbf{a}(q + L) = \mathbf{a}(q), \quad \mathbf{b}(\eta + L) = \mathbf{b}(\eta).$$

It follows that, as in the NG case, the motion of chiral cosmic string loops is periodic, with period $L/2$ [because $\mathbf{x}(t + L/2, \sigma + L/2) = \mathbf{x}(t, \sigma)$]. As noted above, for $k=0$ these loops do not self-intersect and hence are vorton solutions [21]. The majority of studies of vortons to date have assumed that these are circular loops [21] (see however [22–24]). The vortons with $k=0$ found here have entirely arbitrary shapes.

For arbitrary k it is possible to construct solutions of Eqs. (16) just as in the case of NG strings [25–28]. For example, a 1-harmonic loop solution with constant k is given by

$$\mathbf{a}(q) = (\cos q, \sin q, 0), \quad \mathbf{b}(\eta) = (k \cos \eta, -k \sin \eta, 0).$$

This is a circular string oscillating between maximum and minimum radii of $(1+k)/2$ and $(1-k)/2$. (Such a solution was considered numerically in [29] for arbitrary current carrying loops.) This loop never self-intersects for any value of k . Higher order harmonic solutions can also be constructed along very similar lines as Refs. [26–28].

Since loops with $k=0$ never self-intersect, it is interesting to ask how the self-intersection probability of a loop with a given number of harmonics depends on k [or equivalently on the conserved charge N given in Eq. (17)].¹ For simplicity, we will study this question for $k(\phi) = \text{constant}$. In that case, N is given by

$$N = \frac{m}{4} L \sqrt{1 - k^2} \quad (19)$$

where L is the invariant length of the loop. We now show that self-intersecting loop solutions exist for $k > 0$ through the construction of an explicitly self-intersecting loop. Then the probabilities of self-intersection will be studied numerically for loops of a fixed invariant length L but with different numbers of harmonics on them and different values of k .

The condition that a loop self-intersect at time T is that there exists a solution of

$$\mathbf{a}(T + \sigma_1) + \mathbf{b}(T - \sigma_1) = \mathbf{a}(T + \sigma_2) + \mathbf{b}(T - \sigma_2) \quad (20)$$

for some $0 < \sigma_1 \neq \sigma_2 < L$. To show that self-intersection is possible, consider the following solutions for \mathbf{a} and \mathbf{b} that satisfy Eqs. (16):

$$\begin{aligned} \mathbf{a}(q) &= \frac{1}{m} (\cos mq, \sin mq, 0) \\ \mathbf{b}(\eta) &= \frac{k}{n} (\cos n\eta, \cos \chi \sin n\eta, \sin \chi \sin n\eta), \end{aligned} \quad (21)$$

where n and m have no common factors and χ is an arbitrary angle. Now let $c = (\sigma_1 + \sigma_2)/2$, $\delta = (\sigma_1 - \sigma_2)/2$, $q = T + c$ and $\eta = T - c$. Then the self-intersection condition (20) becomes

$$\mathbf{a}(q + \delta) - \mathbf{a}(q - \delta) = \mathbf{b}(\eta + \delta) - \mathbf{b}(\eta - \delta)$$

for which we must find solutions for η, q, δ . On substitution of Eqs. (21), this condition becomes

$$\frac{1}{m} (-\sin mq \sin m\delta, \cos mq \sin m\delta, 0) = \frac{k}{n} (-\sin n\eta \sin n\delta, \cos n\eta \sin n\delta \cos \chi, \cos n\eta \sin n\delta \sin \chi).$$

Hence the requirement is that

$$\cos mq = \cos n\eta = 0 \Leftrightarrow \sin n\eta = \pm 1 = \pm \sin mq, \quad (22)$$

where δ must satisfy

$$\frac{\sin m\delta}{m} = \pm \frac{k}{n} \sin n\delta. \quad (23)$$

Generically, there are solutions to Eqs. (22),(23) and, hence, self-intersections.

¹While initially static NG loops always self-intersect [25], chiral cosmic strings can never be static as observed above.

C. Numerical study of loop self-intersections

More generally one can search for self-intersections numerically and try to determine the self-intersection probability as a function of k and the number of harmonics on the loop. To do this, we used a modified version of the code written by Siemens and Kibble [30] to search for self-intersections of NG loops. These authors built on work of Brown, Convery, and DeLaney [31,32] who devised a method of generating odd harmonic series satisfying a given constraint in terms of products of rotations. The only difference between the NG and chiral cosmic string loops is that for the former, the constraint is $|\dot{\mathbf{b}}|^2 = 1$, while for the latter the constraint is $|\dot{\mathbf{b}}|^2 = k^2 < 1$. Thus here we carry out a simple extension of the work of Siemens and Kibble to study the self-intersection properties of M/P harmonic loops (the

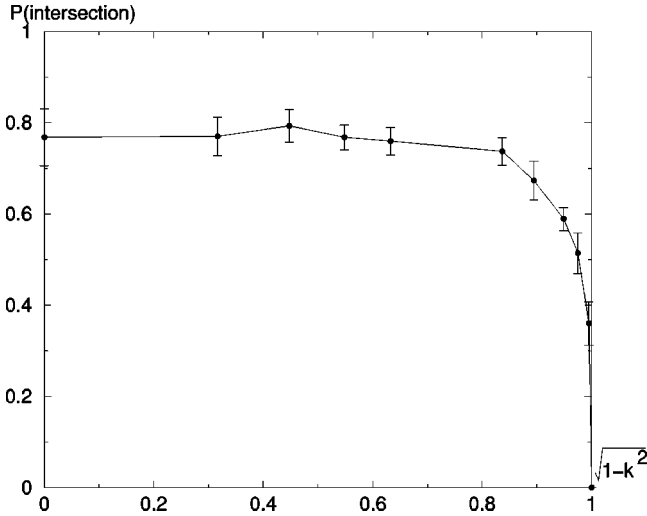


FIG. 1. 5-5 harmonic string.

notation means that there are M harmonics in the solution of **a**, and P in the solution for **b**).

For more technical details on the code, the reader is referred to [30]. In the results presented in Figs. 1–3 below, the rotation angles were given a uniform distribution, with the number of points along the string chosen to be $K = 600$. This gives a resolution of 0.0104712 rad. The cutoff, below which self-intersection was not tested, was taken as 0.084 radians corresponding to 8 step lengths. These are the same parameters as those chosen in [30] which have already been seen to work well. Furthermore, decreasing K or increasing the cutoff did not affect our results.

The self-intersection probability was calculated for M/M cosmic string loops as a function of k [which corresponds to different charges on the loop through Eq. (19)]. Figures 1–3 plot the intersection probability against $\sqrt{1-k^2} \propto N$ for these M/M harmonic loops. (Error bars are one standard deviation.) Notice that for $k=0$ the loops do not self-intersect as was already proved above, whereas for a relatively large range of k the probability is the same as the NG ($k=1$) case. Thus charges on chiral cosmic string loops appear only to

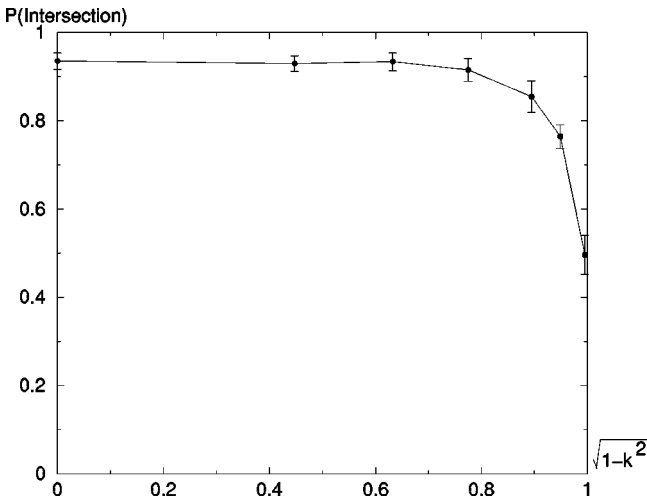


FIG. 2. 11-11 harmonic string.

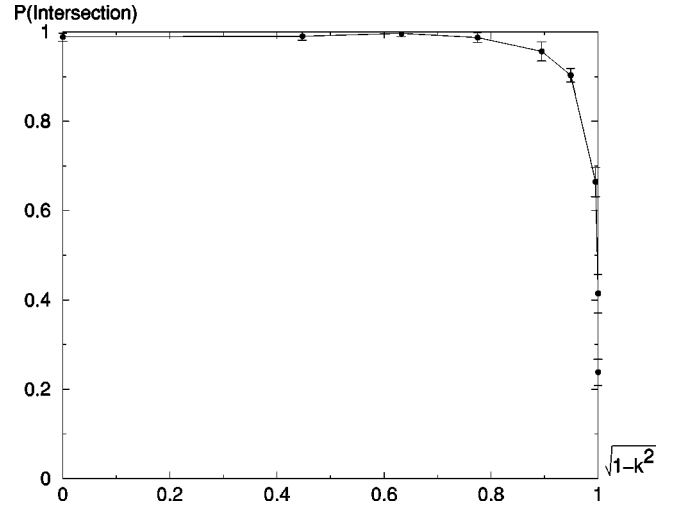


FIG. 3. 25-25 harmonic string.

have a significant effect on the dynamics of the loops when these charges are large. Indeed, if the loops are formed with large charges, they will scarcely ever intersect and this will lead to a cosmological catastrophe since the loops (vortons) will dominate the energy density of the universe. It therefore remains to understand whether or not these charges are expected to be large or small when the loops form: we leave a discussion of this question to the conclusions.

Finally, we note that while the plots show results for M/M harmonic strings, we also ran the code for strings with different numbers of left and right-moving harmonics. This did not substantially change the intersection probability from that of a M/M harmonic string if the harmonics were both close to M .

IV. CONCLUSIONS AND DISCUSSION

The basis for this paper was the action (1) for chiral cosmic strings first proposed in [17]. This is a well-defined, unique, action for strings carrying massless zero modes which travel in one direction along the string at the speed of light. In Sec. II B we reproduced the results of [17] showing how, with suitable gauge choices and treatment of the Lagrange multiplier ψ , the resulting equations of motion are integrable and reduce to the familiar wave equation with two constraints, Eqs. (16). These two constraints are that $|\dot{\mathbf{a}}| = 1$ and $|\dot{\mathbf{b}}| = k^2 \leq 1$. We noted that the reason why $|\dot{\mathbf{b}}|$ lies within the Kibble-Turok sphere rather than on it is that the chiral strings carry a conserved charge (associated with the current on them). In the limit of zero charge, the equations of motion and constraints, Eqs. (16), reduce to those of NG strings in the conformal-temporal gauge as required.

We placed a certain emphasis on gauge choices in Sec. II B since, as we showed in that section, the same action with less appropriate gauge choices can lead to much more complicated equations of motion which are not readily integrable, as for the equations of motion (10) derived in [18].

In Sec. III we showed that chiral current-carrying cosmic strings cannot have cusps on them. Since cosmic rays are predominantly produced at cusps on NG strings [33], it is

likely that a network of chiral cosmic strings will produce fewer ultra-high energy cosmic rays. Unfortunately we are unable to calculate the spectrum of such rays at this time, since this first requires a determination of the evolution of the whole chiral string network: here we have only taken the first steps in determining this evolution.

We also showed that when the charge on the string is maximal (equivalently $k=0$), $\dot{\mathbf{x}}=\mathbf{x}'$, so that the strings move along themselves at half the speed of light and never self-intersect. In the case of loops these correspond to stationary vorton solutions of arbitrary shape. For infinite strings it means that these can never self-intersect to form loops (at least in Minkowski space). Since this is the main mechanism for removing energy from NG string networks, these results suggest that networks of chiral cosmic strings may evolve very differently from NG cosmic string networks.

In another step to study the evolution of chiral cosmic string networks, we considered the self-intersection probability of loops with $0 < k < 1$ and different numbers of harmonics (Sec. III). The results show that only when the charge on the loop is relatively large does the self-intersection probability differ significantly from the NG one.

As a result of this work, we are left with a number of important questions to study in the future. Maybe the most significant one of these is to understand what initial value of k might be expected for the loops and infinite strings formed at the phase transition (which could be, say, at the end of inflation as discussed in the Introduction). If k is initially very small (i.e. the charge on the strings is initially close to being maximal), then chiral cosmic strings are already ruled out, as is the mixed scenario of D -term inflation and strings [15]. The reason is that if the chiral cosmic strings effectively never self-intersect, they rapidly come to dominate the energy density of the universe. Indeed, since the fermions are traveling in one direction only in the chiral case, the current and corresponding charge are larger than in the non-chiral case. Consequently, we would expect the charge to be close to maximal and hence k to be small when the fermion zero modes condense on the string at formation. If, however, the zero modes are formed at a subsequent phase transition, then k is likely to be closer to unity. Indeed, this is the assumption made in [34], where theories giving rise to chiral cosmic strings were constrained by the requirement that they should not over produce vortons. We have arguments suggesting

that k is in fact initially small; these will be presented elsewhere [35].

Here we have restricted attention to loops in which k is constant, but in fact one should also examine the more general case where k is a periodic function of ϕ , though always restricted to the range $0 \leq k \leq 1$. In this case the effective parameter space of loop solutions greatly increases, and we are currently examining how to modify the code used in this paper to study the self-intersections of loops with non-constant k —once again our results will be presented elsewhere [35].

Another objective would be to try to solve Eqs. (16) in a very similar way to the Smith-Vilenkin algorithm which is an exact numerical algorithm for solving the corresponding equations for NG strings in Minkowski space. However, the Smith-Vilenkin algorithm is no longer exact for the chiral string equations: because of the constraint $|\dot{\mathbf{b}}|=k < 1$, the vertices will generally not remain on the lattice as the system evolves. Indeed we believe that there is no value of k for which the algorithm can be made to work—except perhaps $k=0$, a case that is uninteresting in this context as we already know that there the strings are effectively stationary.

Finally, one should also consider to what degree the effects of friction on the evolving chiral cosmic string network are important. Frictional effects on NG and chiral strings are likely to be similar since there are no long range forces in either case; this is unlike the situation for electromagnetically coupled strings [36]. Ultimately the effect of expansion should be incorporated too, though as in the NG case many predictions can be made from Minkowski space results [3]. We are currently studying a number of these questions [35]. Our general conclusion of this paper would be, however, that we have found evidence to suggest that chiral cosmic string networks evolve very differently from NG networks. Hence their cosmological consequences will be very different, and so some caution should be used before simply adding the effects of inflation and NG strings as in [15,16], especially when the specific model under consideration actually produces chiral cosmic strings.

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